GEOMETRIC MEASURE THEORY
OPTIMAL MASS TRANSPORTATION
AND PARTIAL DIFFERENTIAL EQUATIONS

Nonlinear elliptic and parabolic equations with fractional diffusion

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Topics:

1. Fractional Laplacians. Probabilistic meaning and basic properties.
2. Weak solutions and energy. The extension problem.
3. Fully nonlinear integro-differential equations. Introduction to their regularity theory.
4. The fractional Allen-Cahn equation and nonlocal minimal surfaces.
5. Front propagation under fractional diffusion.

In this course I will explain basic ideas and recent developments concerning fractional Laplacians and other nonlocal integro-differential equations. We will start with their basic properties (such as their relation with probability, with fractional Sobolev spaces, and the extension problem associated to them). Later we will explain the basics on regularity theory for fully nonlinear integro-diffrential equations. We will finish treating some nonlinear equations involving these operators: the fractional Allen-Cahn equation, the notion of nonlocal minimal surface, and the fractional Fisher-KPP equation.

The regularity theory and the nonlinear analysis for these equations is one of the most active fields in PDEs nowadays. The main advances have been made in the last decade. Still many remain to be discovered. The interest and applications of these equations is extremaly large, not only within mathematics but mainly in other sciences (which urge for the mathematical study of these objects). Let us mention probability and mathematical finance, image processing, biological invasions and propagations, anomalous fluids, geophysical sciences, etc.