Dirichlet problem at infinity for $\mathcal{A}$-harmonic and minimal graph equations on Cartan-Hadamard manifolds

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I will talk about the recent research on the solvability of the asymptotic Dirichlet problem for a class of quasilinear elliptic PDEs on Cartan-Hadamard manifolds M. These equations include e.g. the Laplace-Beltrami equation and the minimal graph equation

$$\operatorname{div} \frac{\nabla u}{\sqrt{1+|\nabla u|^2}} = 0.$$

We consider Cartan-Hadamard manifolds M whose sectional curvatures are bounded from below and above by certain functions depending on the distance $r = d(\cdot, o)$ to a fixed point $o \in M$. We are, in particular, interested in finding optimal (or close to optimal) curvature upper bounds. In the special cases of the Laplace-Beltrami equation and of the minimal graph equation we are able to solve the asymptotic Dirichlet problem in dimensions $n \geq 3$ if sectional curvatures satisfy

$$-\frac{(\log r(x))^{2\bar{\varepsilon}}}{r(x)^2} \leq K \leq -\frac{1+\varepsilon}{r(x)^2 \, \log r(x)}$$

outside a compact set for some $\varepsilon > \bar{\varepsilon} > 0$. he upper bound is close to optimal since the nonsolvability is known (for the Laplacian) if $K \geq -1/\big(2r(x)^2 \, \log r(x)\big)$. Our results (in the non-rotationally symmetric case) improve on the previously known case of the quadratically decaying upper bound.

In another work (in progress) we also study the asymptotic Dirichlet problem for the minimal graph equation under pointwise pinching condition for the sectional curvatures.

The talk is based on joint works with Jean-Baptiste Casteras, Esko Heinonen, and Jaime Ripoll.