Transport equations and flows for vector fields with borderline integrable divergence

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In the talk, I will talk about transport equations and flows for vector fields with borderline integrable divergence. Under the classical growth condition on the vector fields, we will show that the linear transport Cauchy problem

$$\begin{cases} \dfrac{\partial u}{\partial t} + b \cdot \nabla u = 0 & (0,T) \times \mathbb{R}^n \\ u(0,\cdot) = u_0 \in L^\infty & \mathbb{R}^n. \end{cases}$$

is well-posed if $\mathrm{div}(b)$ enjoys not less than sub-exponential integrability, and is ill-posed if $\mathrm{div}(b)$ is below sub-exponentially integrable. Moreover, if $\mathrm{div}(b)$ is sub-exponentially integrable and b is nearly bounded, we show that there is a non-smooth flow $X(t,x)$ satisfying $\frac{\partial X}{\partial t} = b(t, X(t,x))$ with density functions slightly beyond L^1 integrable. These are joint works with A. Clop, J. Mateu and J. Orobitg.