IRREGULAR SETS FOR RATIOS OF BIRKHOFF AVERAGES ARE RESIDUAL

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Dedicated to Jaume Llibre on the occasion of his 60th birthday

Abstract: It follows from Birkhoff’s Ergodic Theorem that the irregular set of points for which the Birkhoff averages of a given continuous function diverge has zero measure with respect to any finite invariant measure. In strong contrast, for systems with the weak specification property, we show here that if the irregular set is nonempty, then it is residual. This includes topologically transitive topological Markov chains, sofic shifts and more generally shifts with the specification property. We consider also the more general case of ratios of Birkhoff averages of continuous functions and the case when the set of accumulation points of the ratios of Birkhoff averages is a prescribed closed interval. Finally, we give an application of our work to the pointwise dimension of a Gibbs measure on a repeller of a conformal map.

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