BRIOT–BOUQUET’S THEOREM IN HIGH DIMENSION

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Abstract: Let $X$ be a germ of holomorphic vector field at $0 \in \mathbb{C}^n$ and let $E$ be a linear subspace of $\mathbb{C}^n$ which is invariant for the linear part of $X$ at $0$. We give a sufficient condition that imply the existence of a non-singular invariant manifold tangent to $E$ at $0$. It generalizes to higher dimensions the conditions in the classical Briot–Bouquet’s Theorem: roughly speaking, we impose that the convex hull of the eigenvalues $\mu_i$ corresponding to $E$ does not contain 0 and there are no resonances between the $\mu_i$ and the complementary eigenvalues. As an application, we propose an elementary proof of the analyticity of the local stable and unstable manifolds of a real analytic vector field at a singular point.

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