TOPOLOGICAL AND POLYNOMIAL INVARIANTS,
MODULI SPACES, IN CLASSIFICATION PROBLEMS
OF POLYNOMIAL VECTOR FIELDS

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Dedicated to J. Llibre on his 60th birthday

Abstract: We describe the origin and evolution of ideas on topological and polynomial invariants and their interaction, in problems of classification of polynomial vector fields. The concept of moduli space is discussed in the last section and we indicate its value in understanding the dynamics of families of such systems. Our interest here is in the concepts and the way they interact in the process of topologically classifying polynomial vector fields. We survey the literature giving an ample list of references and we illustrate the ideas on the testing ground of families of quadratic vector fields. In particular, the role of polynomial invariants is illustrated in the proof of our theorem in the section next to last. These concepts have proven their worth in a number of classification results, among them the most recent work on the geometric classification of the whole class of quadratic vector fields, according to their configurations of infinite singularities. An analog work including both finite and infinite singularities of the whole quadratic class, joint work with J. C. Artés, J. Llibre, and N. Vulpe, is in progress.

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