ON THE STRUCTURE OF THE INTERSECTION OF TWO MIDDLE THIRD CANTOR SETS

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Abstract _

Motivated by the study of planar homoclinic bifurcations, in this paper we describe how the intersection of two middle third Cantor sets changes as the sets are translated across each other. The resulting description shows that the intersection is never empty; in fact, the intersection can be either finite or infinite in size. We show that when the intersection is finite then the number of points in the intersection will be either 2^n or $3 \cdot 2^n$. We also explore the Hausdorff dimension of the intersection of two middle third Cantor sets as the sets are translated across one another. We show that the Hausdorff dimension of the intersection can take on any value from 0 to $\ln 2/\ln 3$; in addition, we show that for each Hausdorff dimension, between 0 and $\ln 2/\ln 3$, there is a dense set of translation parameters for which the intersections have that particular Hausdorff dimension.