WEIGHTED $L_p$ SPACES
AND POINTWISE ERGODIC THEOREMS

RYOTARO SATO

Abstract
In this paper we give an operator theoretic version of a recent result of F. J. Martín-Reyes and A. de la Torre concerning the problem of finding necessary and sufficient conditions for a nonsingular point transformation to satisfy the Pointwise Ergodic Theorem in $L_p$. We consider a positive conservative contraction $T$ on $L_1$ of a $\sigma$-finite measure space $(X, \mathcal{F}, \mu)$, a fixed function $e$ in $L_1$ with $e > 0$ on $X$, and two positive measurable functions $V$ and $W$ on $X$. We then characterize the pairs $(V, W)$ such that for any $f$ in $L_p(V \, d\mu)$ the averages

$$R^n_0(f, e) = \frac{\left( \sum_{k=0}^n T^k f \right)}{\left( \sum_{k=0}^n T^k e \right)}$$

converge almost everywhere to a function in $L_p(W \, d\mu)$. The characterizations are given for all $p$, $1 \leq p < \infty$. 