NEW CHARACTERIZATIONS OF VON NEUMANN REGULAR RINGS AND A CONJECTURE OF SHAMSUDDIN

CARL FAITH

Abstract

A theorem of Utumi states that if $R$ is a right self-injective ring such that every maximal ideal has nonzero annihilator, then $R$ modulo the Jacobson radical $J$ is a finite product of simple rings and is a von Neuman regular ring. We prove two theorems and a conjecture of Shamsuddin that characterize when $R$ itself is a von Neumann ring, using a splitting theorem of the author on when the maximal regular ideal of a ring splits off.