COMPOSITION OF MAXIMAL OPERATORS

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Abstract

Consider the Hardy-Littlewood maximal operator

\[ Mf(x) = \sup_{Q \ni x} \frac{1}{|Q|} \int_Q |f(y)| \, dy. \]

It is known that \( M \) applied to \( f \) twice is pointwise comparable to the maximal operator \( M_{L \log L} f \), defined by replacing the mean value of \( |f| \) over the cube \( Q \) by the \( L \log L \)-mean, namely

\[ M_{L \log L} f(x) = \sup_{x \in Q} \frac{1}{|Q|} \int_Q |f(y)| \log \left( e + \frac{|f|}{|f|_Q} \right) (y) \, dy, \]

where \( |f|_Q = \frac{1}{|Q|} \int_Q |f| \) (see [L], [LN], [P]).

In this paper we prove that, more generally, if \( \Phi(t) \) and \( \Psi(t) \) are two Young functions, there exists a third function \( \Theta(t) \), whose explicit form is given as a function of \( \Phi(t) \) and \( \Psi(t) \), such that the composition \( M_{\Phi} \circ M_{\Psi} \) is pointwise comparable to \( M_{\Theta} \). Through the paper, given an Orlicz function \( A(t) \), by \( M_A f \) we mean

\[ M_A f(x) = \sup_{Q \ni x} ||f||_{A,Q} \]

where \( ||f||_{A,Q} = \inf \left\{ \lambda > 0 : \frac{1}{|Q|} \int_Q A \left( \frac{|f|}{\lambda} \right) (x) \, dx \leq 1 \right\} \).

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