UNIQUENESS AND EXISTENCE OF SOLUTIONS IN THE $BV_t(Q)$ SPACE TO A DOUBLY NONLINEAR PARABOLIC PROBLEM

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Abstract

In this paper we present some results on the uniqueness and existence of a class of weak solutions (the so called BV solutions) of the Cauchy-Dirichlet problem associated to the doubly nonlinear diffusion equation

$$b(u)_t - \text{div}( |\nabla u - k(b(u))e|^{p-2}(\nabla u - k(b(u))e)) + g(x,u) = f(t,x).$$

This problem arises in the study of some turbulent regimes: flows of incompressible turbulent fluids through porous media, gases flowing in pipelines, etc. The solvability of this problem is established in the $BV_t(Q)$ space. We prove some comparison properties (implying uniqueness) when the set of jumping points of the BV solution has $N$-dimensional null measure and suitable additional conditions as, for instance, $b^{-1}$ locally Lipschitz. The existence of this type of weak solution is based on suitable uniform estimates of the BV norm of an approximated solution.

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