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WEIGHTED NORM INEQUALITIES FOR THE GEOMETRIC MAXIMAL OPERATOR

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Abstract

We consider two closely related but distinct operators,

$$M_0 f(x) = \sup_{I \ni x} \exp\left(\frac{1}{|I|}\int_I \log|f|\,dy\right) \quad \text{and}$$
$$M_0^* f(x) = \lim_{r\to 0} \sup_{I \ni x} \left(\frac{1}{|I|}\int_I |f|^r\,dy\right)^{1/r}.$$

We give sufficient conditions for the two operators to be equal and show that these conditions are sharp. We also prove two-weight, weighted norm inequalities for both operators using our earlier results about weighted norm inequalities for the minimal operator:

$$\mathcal{m} f(x) = \inf_{I \ni x} \frac{1}{|I|}\int_I |f|\,dy.$$

This extends the work of X. Shi; H. Wei, S. Xianliang and S. Qiyu; X. Yin and B. Muckenhoupt; and C. Sbordone and I. Wik.

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