

INVARIANT SUBSPACES ON MULTIPLY CONNECTED DOMAINS

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Abstract

The lattice of invariant subspaces of several Banach spaces of analytic functions on the unit disk, for example the Bergman spaces and the Dirichlet spaces, have been studied recently. A natural question is to what extent these investigations carry over to analogously defined spaces on an annulus. We consider this question in the context of general Banach spaces of analytic functions on finitely connected domains Ω . The main result reads as follows: Assume that B is a Banach space of analytic functions satisfying some conditions on the domain Ω . Assume further that $M(B)$ is the set of all multipliers of B . Let Ω_1 be a domain obtained from Ω by adding some of the bounded connectivity components of $\mathbb{C} \setminus \Omega$. Also, let B_1 be the closed subspace of B of all functions that extend analytically to Ω_1 . Then the mapping $I \mapsto \text{clos}(I \cdot M(B))$ gives a one-to-one correspondence between a class of multiplier invariant subspaces I of B_1 , and a class of multiplier invariant subspaces J of B . The inverse mapping is given by $J \mapsto J \cap B_1$.

Keywords. Banach spaces of analytic functions, multiplier invariant subspace, index, linear operator, spectrum, holomorphic functional calculus.

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