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ANALYTIC CAPACITY, CALDERÓN-ZYGMUND OPERATORS, AND RECTIFIABILITY

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Abstract

For $K \subset \mathbb{C}$ compact, we say that K has vanishing analytic capacity (or $\gamma(K) = 0$) when all bounded analytic functions on $\mathbb{C} \backslash K$ are constant. We would like to characterize $\gamma(K) = 0$ geometrically. Easily, $\gamma(K) > 0$ when K has Hausdorff dimension larger than 1, and $\gamma(K) = 0$ when $\dim(K) < 1$. Thus only the case when $\dim(K) = 1$ is interesting. So far there is no characterization of $\gamma(K) = 0$ in general, but the special case when the Hausdorff measure $H^1(K)$ is finite was recently settled. In this case, $\gamma(K) = 0$ if and only if K is unrectifiable (or Besicovitch-irregular), i.e., if $H^1(K \cap \Gamma) = 0$ for all C^1-curves Γ, as was conjectured by Vitushkin.

In the present text, we try to explain the structure of the proof of this result, and present the necessary techniques. These include the introduction to Menger curvature in this context (by M. Melnikov and co-authors), and the important use of geometric measure theory (results on quantitative rectifiability), but we insist most on the role of Calderón-Zygmund operators and $T(b)$-Theorems.