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REGULAR MAPPINGS BETWEEN DIMENSIONS

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Abstract

The notions of Lipschitz and bilipschitz mappings provide classes of mappings connected to the geometry of metric spaces in certain ways. A notion between these two is given by "regular mappings" (reviewed in Section 1), in which some non-bilipschitz behavior is allowed, but with limitations on this, and in a quantitative way. In this paper we look at a class of mappings called (s,t)-*regular mappings*. These mappings are the same as ordinary regular mappings when $s = t$, but otherwise they behave somewhat like projections. In particular, they can map sets with Hausdorff dimension s to sets of Hausdorff dimension t. We mostly consider the case of mappings between Euclidean spaces, and show in particular that if $f\colon \mathbf{R}^s \to \mathbf{R}^n$ is an (s,t)-regular mapping, then for each ball B in $\mathbf{R}^s$ there is a linear mapping $\lambda\colon \mathbf{R}^s \to \mathbf{R}^{s-t}$ and a subset E of B of substantial measure such that the pair (f,λ) is bilipschitz on E. We also compare these mappings in comparison with "nonlinear quotient mappings" from [**6**].

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