REPRESENTATION OF ALGEBRAIC DISTRIBUTIVE LATTICES WITH $\aleph_1$ COMPACT ELEMENTS AS IDEAL LATTICES OF REGULAR RINGS

FRIEDRICH WEHRUNG

Abstract

We prove the following result:

Theorem. Every algebraic distributive lattice $D$ with at most $\aleph_1$ compact elements is isomorphic to the ideal lattice of a von Neumann regular ring $R$.

(By earlier results of the author, the $\aleph_1$ bound is optimal.) Therefore, $D$ is also isomorphic to the congruence lattice of a sectionally complemented modular lattice $L$, namely, the principal right ideal lattice of $R$. Furthermore, if the largest element of $D$ is compact, then one can assume that $R$ is unital, respectively, that $L$ has a largest element. This extends several known results of G. M. Bergman, A. P. Huhn, J. Tůma, and of a joint work of G. Grätzer, H. Lakser, and the author, and it solves Problem 2 of the survey paper [10].

The main tool used in the proof of our result is an amalgamation theorem for semilattices and algebras (over a given division ring), a variant of previously known amalgamation theorems for semilattices and lattices, due to J. Tůma, and G. Grätzer, H. Lakser, and the author.

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