

**L^p BOUNDS FOR RIESZ TRANSFORMS AND SQUARE
ROOTS ASSOCIATED TO SECOND ORDER ELLIPTIC
OPERATORS**

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Abstract

We consider the Riesz transforms $\nabla L^{-1/2}$, where $L \equiv -\operatorname{div} A(x) \nabla$, and A is an accretive, $n \times n$ matrix with bounded measurable complex entries, defined on \mathbb{R}^n . We establish boundedness of these operators on $L^p(\mathbb{R}^n)$, for the range $p_n < p \leq 2$, where $p_n = 2n/(n+2)$, $n \geq 2$, and we obtain a weak-type estimate at the endpoint p_n . The case $p = 2$ was already known: it is equivalent to the solution of the square root problem of T. Kato.

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