BOUNDEDNESS OF THE WEYL FRACTIONAL INTEGRAL ON ONE-SIDED WEIGHTED LEBESGUE AND LIPSCHITZ SPACES

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Abstract

In this paper we introduce the one-sided weighted spaces $L_w^-(\beta)$, $-1 < \beta < 1$. The purpose of this definition is to obtain an extension of the Weyl fractional integral operator $I^+_\alpha$ from $L^p_w$ into a suitable weighted space. Under certain condition on the weight $w$, we have that $L_w^-(0)$ coincides with the dual of the Hardy space $H^1_w$. We prove for $0 < \beta < 1$, that $L_w^-(\beta)$ consists of all functions satisfying a weighted Lipschitz condition. In order to give another characterization of $L_w^- (\beta)$, $0 \leq \beta < 1$, we also prove a one-sided version of John-Nirenberg Inequality.

Finally, we obtain necessary and sufficient conditions on the weight $w$ for the boundedness of an extension of $I^+_\alpha$ from $L^p_w$ into $L_w^- (\beta)$, $-1 < \beta < 1$, and its extension to a bounded operator from $L_w^-(0)$ into $L_w^- (\alpha)$.

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Key words. Weyl fractional integral, weights, weighted Lebesgue and Lipschitz spaces, weighted BMO.

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