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ON L^p ESTIMATES FOR SQUARE ROOTS OF SECOND ORDER ELLIPTIC OPERATORS ON $\mathbb{R}^n$

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Abstract

We prove that the square root of a uniformly complex elliptic operator $L = -\operatorname{div}(A\nabla)$ with bounded measurable coefficients in $\mathbb{R}^n$ satisfies the estimate $\|L^{1/2}f\|_p \lesssim \|\nabla f\|_p$ for $\sup(1, \frac{2n}{n+4} - \varepsilon) < p < \frac{2n}{n-2} + \varepsilon$, which is new for $n \geq 5$ and $p < 2$ or for $n \geq 3$ and $p > \frac{2n}{n-2}$. One feature of our method is a Calderón-Zygmund decomposition for Sobolev functions. We make some further remarks on the topic of the converse L^p inequalities (i.e. Riesz transforms bounds), pushing the recent results of [**BK2**] and [**HM**] for $\frac{2n}{n+2} < p < 2$ when $n \geq 3$ to the range $\sup(1, \frac{2n}{n+2} - \varepsilon) < p < 2+\varepsilon'$. In particular, we obtain that $L^{1/2}$ extends to an isomorphism from $\dot{W}^{1,p}(\mathbb{R}^n)$ to $L^p(\mathbb{R}^n)$ for p in this range. We also generalize our method to higher order operators.

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