

**CARLESON'S THEOREM:
PROOF, COMPLEMENTS, VARIATIONS**

MICHAEL T. LACEY

Abstract

Carleson's Theorem from 1965 states that the partial Fourier sums of a square integrable function converge pointwise. We prove an equivalent statement on the real line, following the method developed by the author and C. Thiele. This theorem, and the proof presented, is at the center of an emerging theory which complements the statement and proof of Carleson's theorem. An outline of these variations is also given.

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