INVolvIng Symmetries of RIEMANN surfACeS TO A Study oF THE Mapping CLAss GRouP

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Abstract

A pair of symmetries \((\sigma, \tau)\) of a Riemann surface \(X\) is said to be perfect if their product belongs to the derived subgroup of the group \(\text{Aut}^+(X)\) of orientation preserving automorphisms. We show that given \(g \neq 2, 3, 5, 7\) there exists a Riemann surface \(X\) of genus \(g\) admitting a perfect pair of symmetries of certain topological type. On the other hand we show that a twist can be written as a product of two symmetries of the same type which leads to a decomposition of a twist as a product of two commutators: one from \(M'\) which entirely lives on a Riemann surface and one from \(M^{\pm}\). As a result we obtain the perfectness of the mapping class group \(M_g\) for such \(g\) relying only on results of Birman [1] but not on influential paper of Powell [6] nor on Johnson’s rediscovery of Dehn lantern relation [3] and nor on recent results of Korkmaz-Ozbagci [4] who found explicit presentation of a twist as a product of two commutators.

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