EXTRAPOLATION AND SHARP NORM ESTIMATES FOR CLASSICAL OPERATORS ON WEIGHTED LEBESGUE SPACES

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Abstract _

We obtain sharp weighted L^p estimates in the Rubio de Francia extrapolation theorem in terms of the A_p characteristic constant of the weight. Precisely, if for a given $1 < r < \infty$ the norm of a sublinear operator on $L^r(w)$ is bounded by a function of the A_r characteristic constant of the weight w, then for p > r it is bounded on $L^p(v)$ by the same increasing function of the A_p characteristic constant of v, and for p < r it is bounded on $L^p(v)$ by the same increasing function of the A_p characteristic constant of v. For some operators these bounds are sharp, but not always. In particular, we show that they are sharp for the Hilbert, Beurling, and martingale transforms.

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Key words. Extrapolation, sharp weighted estimates, dyadic square function, dyadic paraproduct, martingale transform, Hilbert transform, Beurling transform.

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