

**LIPSCHITZ SPACES AND CALDERÓN-ZYGMUND
OPERATORS ASSOCIATED TO NON-DOUBLING
MEASURES**

JOSÉ GARCÍA-CUERVA AND A. EDUARDO GATTO

Abstract

In the setting of a metric measure space (\mathbb{X}, d, μ) with an n -dimensional Radon measure μ , we give a necessary and sufficient condition for the boundedness of Calderón-Zygmund operators associated to the measure μ on Lipschitz spaces on the support of μ . Also, for the Euclidean space \mathbb{R}^d with an arbitrary Radon measure μ , we give several characterizations of Lipschitz spaces on the support of μ , $\mathcal{L}ip(\alpha, \mu)$, in terms of mean oscillations involving μ . This allows us to view the “regular” BMO space of X. Tolsa as a limit case for $\alpha \rightarrow 0$ of the spaces $\mathcal{L}ip(\alpha, \mu)$.

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