ON SOME MEAN OSCILLATION INEQUALITIES FOR MARTINGALES

MASATO KIKUCHI

Abstract _____

Let $(X, \|\cdot\|_X)$ be a Banach function space over a nonatomic probability space $(\Omega, \Sigma, \mathbb{P})$. If $f = (f_n)_{n \in \mathbb{Z}_+}$ is a martingale with respect to a filtration $\mathcal{F} = (\mathcal{F}_n)_{n \in \mathbb{Z}_+}$, then we define

$$\theta_{\mathcal{F}}f = \sup_{0 \le n \le m < \infty} \mathbb{E}\big[|f_m - f_{n-1}| \, \big| \, \mathcal{F}_n \big],$$

where $f_{-1} \equiv 0$. In this paper, we give a necessary and sufficient condition for the existence of constants c and C such that for any martingale $f = (f_n)_{n \in \mathbb{Z}_+}$,

$$c \lim_{n \to \infty} \|f_n\|_X \le \|\theta_{\mathcal{F}} f\|_X \le C \lim_{n \to \infty} \|f_n\|_X.$$

²⁰⁰⁰ Mathematics Subject Classification. 60G42, 60G46, 46E30.

Key words. Martingale, Banach function space, rearrangement-invariant function space, Boyd index.

This research was partially supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid for Scientific Research, No. 14540164, 2004.