

ASYMPTOTIC ISOPERIMETRY OF BALLS IN METRIC MEASURE SPACES

ROMAIN TESSERA

Abstract

In this paper, we study the asymptotic behavior of the volume of spheres in metric measure spaces. We first introduce a general setting adapted to the study of asymptotic isoperimetry in a general class of metric measure spaces. Let \mathcal{A} be a family of subsets of a metric measure space (X, d, μ) , with finite, unbounded volume. For $t > 0$, we define

$$I_{\mathcal{A}}^{\downarrow}(t) = \inf_{A \in \mathcal{A}, \mu(A) \geq t} \mu(\partial A).$$

We say that \mathcal{A} is asymptotically isoperimetric if $\forall t > 0$

$$I_{\mathcal{A}}^{\downarrow}(t) \leq CI(Ct),$$

where I is the profile of X . We show that there exist graphs with uniform polynomial growth whose balls are not asymptotically isoperimetric and we discuss the stability of related properties under quasi-isometries. Finally, we study the asymptotically isoperimetric properties of connected subsets in a metric measure space. In particular, we build graphs with uniform polynomial growth whose connected subsets are not asymptotically isoperimetric.

2000 *Mathematics Subject Classification*. Primary: 53C23; Secondary: 05C99.

Key words. Isoperimetry, large-scale geometry, balls, metric measure spaces.