INTRINSIC GEOMETRY ON THE CLASS OF
PROBABILITY DENSITIES AND EXPONENTIAL
FAMILIES

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Abstract

We present a way of thinking of exponential families as geodesic surfaces in the class of positive functions considered as a (multiplicative) sub-group $G^+$ of the group $G$ of all invertible elements in the algebra $\mathcal{A}$ of all complex bounded functions defined on a measurable space. For that we have to study a natural geometry on that algebra. The class $\mathcal{D}$ of densities with respect to a given measure will happen to be representatives of equivalence classes defining a projective space in $\mathcal{A}$. The natural geometry is defined by an intrinsic group action which allows us to think of the class of positive, invertible functions $G^+$ as a homogeneous space. Also, the parallel transport in $G^+$ and $\mathcal{D}$ will be given by the original group action. Besides studying some relationships among these constructions, we examine some Riemannian geometries and provide a geometric interpretation of Pinsker’s and other classical inequalities. Also we provide a geometric reinterpretation of some relationships between polynomial sequences of convolution type, probability distributions on $N$ in terms of geodesics in the Banach space $\ell_1(\alpha)$.

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