WEIGHTED NORM INEQUALITIES FOR CALDERÓN-ZYGMUND OPERATORS WITHOUT DOUBLING CONDITIONS

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Abstract

Let $\mu$ be a Borel measure on $\mathbb{R}^d$ which may be non doubling. The only condition that $\mu$ must satisfy is $\mu(B(x, r)) \leq Cr^n$ for all $x \in \mathbb{R}^d$, $r > 0$ and for some fixed $n$ with $0 < n \leq d$. In this paper we introduce a maximal operator $N$, which coincides with the maximal Hardy-Littlewood operator if $\mu(B(x, r)) \approx r^n$ for $x \in \text{supp}(\mu)$, and we show that all $n$-dimensional Calderón-Zygmund operators are bounded on $L^p(w \, d\mu)$ if and only if $N$ is bounded on $L^p(w \, d\mu)$, for a fixed $p \in (1, \infty)$. Also, we prove that this happens if and only if some conditions of Sawyer type hold. We obtain analogous results about the weak $(p, p)$ estimates.

This type of weights do not satisfy a reverse Hölder inequality, in general, but some kind of self improving property still holds. On the other hand, if $f \in \text{RBMO}(\mu)$ and $\varepsilon > 0$ is small enough, then $e^{\varepsilon f}$ belongs to this class of weights.

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Key words. Calderón-Zygmund operators, weights, non doubling measures.

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