Let $V$ be a vector space over a field $F$. If $G \leq GL(V, F)$, the central dimension of $G$ is the $F$-dimension of the vector space $V/C_V(G)$. In [DEK] and [KS], soluble linear groups in which the set $L_{icd}(G)$ of all proper infinite central dimensional subgroups of $G$ satisfies the minimal condition and the maximal condition, respectively, have been described. On the other hand, in [MOS], periodic locally radical linear groups in which $L_{icd}(G)$ satisfies one of the weak chain conditions (the weak minimal condition or the weak maximal condition) have been characterized. In this paper, we begin the study of the non-periodic case by describing locally nilpotent linear groups in which $L_{icd}(G)$ satisfies one of the two weak chain conditions.