FAITHFUL LINEAR REPRESENTATIONS OF BANDS

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Abstract

A band is a semigroup consisting of idempotents. It is proved that for any field $K$ and any band $S$ with finitely many components, the semigroup algebra $K[S]$ can be embedded in upper triangular matrices over a commutative $K$-algebra. The proof of a theorem of Malcev [4, Theorem 10] on embeddability of algebras into matrix algebras over a field is corrected and it is proved that if $S = F \cup E$ is a band with two components $E$, $F$ such that $F$ is an ideal of $S$ and $E$ is finite, then $S$ is a linear semigroup. Certain sufficient conditions for linearity of a band $S$, expressed in terms of annihilators associated to $S$, are also obtained.


Key words. Linear band, semigroup algebra, triangular matrices, annihilator, PI rings, normal band.

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