A CHOICE OF SOBOLEV SPACES ASSOCIATED WITH ULTRASPHERICAL EXPANSIONS

JORGE J. BETANCOR, JUAN C. FARIÑA, LOURDES RODRÍGUEZ-MESA, RICARDO TESTONI, AND JOSÉ L. TORREÁ

Dedicated to the memory of Professor Carlos Segovia

Abstract

We discuss two possible definitions for Sobolev spaces associated with ultraspherical expansions. These definitions depend on the notion of higher order derivative. We show that in order to have an isomorphism between Sobolev and potential spaces, the higher order derivatives to be considered are not the iteration of the first order derivatives. Some discussions about higher order Riesz transforms are involved. Also we prove that the maximal operator for the Poisson integral in the ultraspherical setting is bounded on the Sobolev spaces.

2000 Mathematics Subject Classification. Primary: 42C05; Secondary: 42C15.

Key words. Sobolev spaces, ultraspherical expansions.

Partially supported by MTM2007/65699, MTM 2008-06621-C02-01 and PCI2006-A7-0670.