Suppose $N$ is a nice subgroup of the primary abelian group $G$ and $A = G/N$. The paper discusses various contexts in which $G$ satisfying some property implies that $A$ also satisfies the property, or visa versa, especially when $N$ is countable. For example, if $n$ is a positive integer, $G$ has length not exceeding $\omega_1$ and $N$ is countable, then $G$ is $n$-summable iff $A$ is $n$-summable. When $A$ is separable and $N$ is countable, we discuss the condition that any such $G$ decomposes into the direct sum of a countable and a separable group, and we show that it is undecidable in ZFC whether this condition implies that $A$ must be a direct sum of cyclics. We also relate these considerations to the study of nice bases for primary abelian groups.

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