POLYNOMIAL DIFFERENTIAL EQUATIONS WITH MANY REAL OVALS IN THE SAME ALGEBRAIC COMPLEX SOLUTION

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Abstract

Let Fol_{\mathbb{R}}(2, d) be the space of real algebraic foliations of degree d in \mathbb{RP}(2). For fixed d, let Int_{\mathbb{R}}(2, d) = \{F \in Fol_{\mathbb{R}}(2, d) \mid F \text{ has a non-constant rational first integral}\}. Given F \in Int_{\mathbb{R}}(2, d), with primitive first integral G, set O(F) = number of real ovals of the generic level (G = c). Let O(d) = \sup\{O(F) \mid F \in Int_{\mathbb{R}}(2, d)\}.

The main purpose of this paper is to prove that O(d) = +\infty for all d \geq 5.

2010 Mathematics Subject Classification. 34C09, 34C08, 37F75.

Key words. Real ovals, differential equations.