SHARP NORM INEQUALITIES FOR COMMUTATORS OF CLASSICAL OPERATORS

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Abstract: We prove several sharp weighted norm inequalities for commutators of classical operators in harmonic analysis. We find sufficient A_p-bump conditions on pairs of weights (u, v) such that $[b, T]$, $b \in BMO$ and T a singular integral operator (such as the Hilbert or Riesz transforms), maps $L^p(v)$ into $L^p(u)$. Because of the added degree of singularity, the commutators require a "double log bump" as opposed to that of singular integrals, which only require single log bumps. For the fractional integral operator I_α we find the sharp one-weight bound on $[b, I_\alpha]$, $b \in BMO$, in terms of the $A_{p,q}$ constant of the weight. We also prove sharp two-weight bounds for $[b, I_\alpha]$ analogous to those of singular integrals. We prove two-weight weak type inequalities for $[b, T]$ and $[b, I_\alpha]$ for pairs of factored weights. Finally we construct several examples showing our bounds are sharp.

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