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ENTROPY AND FLATNESS IN LOCAL ALGEBRAIC DYNAMICS

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Abstract: For a local endomorphism of a noetherian local ring we introduce a notion of entropy, along with two other asymptotic invariants. We use this notion of entropy to extend numerical conditions in Kunz' regularity criterion to every contracting endomorphism of a noetherian local ring, and to give a characteristic-free interpretation of the definition of Hilbert-Kunz multiplicity. We also show that every finite endomorphism of a complete noetherian local ring of equal characteristic can be lifted to a finite endomorphism of a complete regular local ring. The local ring of an algebraic or analytic variety at a point fixed by a finite self-morphism inherits a local endomorphism whose entropy is well-defined. This situation arises at the vertex of the affine cone over a projective variety with a polarized self-morphism, where we compare entropy with degree.

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