

A NEW CHARACTERIZATION OF TRIEBEL-LIZORKIN SPACES ON $\mathbb{R}^n$

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Abstract: In this paper, the authors characterize the Triebel-Lizorkin space $\dot{F}_{p,q}^\alpha(\mathbb{R}^n)$ via a new square function

$$S_{\alpha,q}(f)(x) = \left\{ \sum_{k \in \mathbb{Z}} 2^{k\alpha q} \left| \frac{1}{|B(x, 2^{-k})|} \int_{B(x, 2^{-k})} [f(x) - f(y)] dy \right|^q \right\}^{1/q},$$

where $f \in L_{\text{loc}}^1(\mathbb{R}^n) \cap \mathcal{S}'(\mathbb{R}^n)$, $x \in \mathbb{R}^n$, $\alpha \in (0, 2)$ and $p, q \in (1, \infty]$. Similar characterizations are also established for Triebel-Lizorkin spaces $\dot{F}_{p,q}^\alpha(\mathbb{R}^n)$ with $\alpha \in (0, \infty) \setminus 2\mathbb{N}$ and $p, q \in (1, \infty]$, and for Besov spaces $\dot{B}_{p,q}^\alpha(\mathbb{R}^n)$ with $\alpha \in (0, \infty) \setminus 2\mathbb{N}$, $p \in (1, \infty]$ and $q \in (0, \infty]$.

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