

ON RINGS WHOSE MODULES HAVE NONZERO HOMOMORPHISMS TO NONZERO SUBMODULES

Y. TOLOOEI AND M. R. VEDADI

Abstract: We carry out a study of rings R for which $\text{Hom}_R(M, N) \neq 0$ for all nonzero $N \leq M_R$. Such rings are called retractable. For a retractable ring, Artinian condition and having Krull dimension are equivalent. Furthermore, a right Artinian ring in which prime ideals commute is precisely a right Noetherian retractable ring. Retractable rings are characterized in several ways. They form a class of rings that properly lies between the class of pseudo-Frobenius rings, and the class of max divisible rings for which the converse of Schur's lemma holds. For several types of rings, including commutative rings, retractability is equivalent to semi-Artinian condition. We show that a Köthe ring R is an Artinian principal ideal ring if and only if it is a certain retractable ring, and determine when R is retractable.

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