POTENTIAL MAPS, HARDY SPACES, AND TENT SPACES ON SPECIAL LIPSCHITZ DOMAINS

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Abstract: Suppose that $\Omega$ is the open region in $\mathbb{R}^n$ above a Lipschitz graph and let $d$ denote the exterior derivative on $\mathbb{R}^n$. We construct a convolution operator $T$ which preserves support in $\Omega$, is smoothing of order 1 on the homogeneous function spaces, and is a potential map in the sense that $dT$ is the identity on spaces of exact forms with support in $\Omega$. Thus if $f$ is exact and supported in $\Omega$, then there is a potential $u$, given by $u = Tf$, of optimal regularity and supported in $\Omega$, such that $du = f$. This has implications for the regularity in homogeneous function spaces of the de Rham complex on $\Omega$ with or without boundary conditions. The operator $T$ is used to obtain an atomic characterisation of Hardy spaces $H^p$ of exact forms with support in $\Omega$ when $n/(n+1) < p \leq 1$. This is done via an atomic decomposition of functions in the tent spaces $T^p(\mathbb{R}^n \times \mathbb{R}^+)$ with support in a tent $T(\Omega)$ as a sum of atoms with support away from the boundary of $\Omega$. This new decomposition of tent spaces is useful, even for scalar valued functions.

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