Publ. Mat. **58** (2014), 133–154
DOI: 10.5565/PUBLMAT_58114_06

WEAK AND VISCOSITY SOLUTIONS OF THE FRACTIONAL LAPLACE EQUATION

Raffaella Servadei and Enrico Valdinoci

Abstract: Aim of this paper is to show that weak solutions of the following fractional Laplacian equation

$$\begin{cases} (-\Delta)^s u = f & \text{in } \Omega \\ u = g & \text{in } \mathbb{R}^n \setminus \Omega \end{cases}$$

are also continuous solutions (up to the boundary) of this problem in the viscosity sense.

Here $s \in (0,1)$ is a fixed parameter, Ω is a bounded, open subset of $\mathbb{R}^n$ ($n \geqslant 1$) with C^2-boundary, and $(-\Delta)^s$ is the fractional Laplacian operator, that may be defined as

$$(-\Delta)^s u(x) := c(n,s) \int_{\mathbb{R}^n} \frac{2u(x) - u(x+y) - u(x-y)}{|y|^{n+2s}}\, dy,$$

for a suitable positive normalizing constant $c(n,s)$, depending only on n and s.

In order to get our regularity result we first prove a maximum principle and then, using it, an interior and boundary regularity result for weak solutions of the problem.

As a consequence of our regularity result, along the paper we also deduce that the first eigenfunction of $(-\Delta)^s$ is strictly positive in Ω.

2010 Mathematics Subject Classification: 35R09, 45K05, 49N60, 35D30.

Key words: Integrodifferential operators, fractional Laplacian, weak solutions, viscosity solutions, regularity theory.