WEAK AND VISCOSITY SOLUTIONS OF THE FRACTIONAL LAPLACE EQUATION

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Abstract: Aim of this paper is to show that weak solutions of the following fractional Laplacian equation

\[
\begin{cases}
(-\Delta)^s u = f & \text{in } \Omega \\
u = g & \text{in } \mathbb{R}^n \setminus \Omega
\end{cases}
\]

are also continuous solutions (up to the boundary) of this problem in the viscosity sense.

Here \( s \in (0,1) \) is a fixed parameter, \( \Omega \) is a bounded, open subset of \( \mathbb{R}^n \) (\( n \geq 1 \)) with \( C^2 \)-boundary, and \( (-\Delta)^s \) is the fractional Laplacian operator, that may be defined as

\[
(-\Delta)^s u(x) := c(n, s) \int_{\mathbb{R}^n} \frac{2u(x) - u(x + y) - u(x - y)}{|y|^{n+2s}} \, dy,
\]

for a suitable positive normalizing constant \( c(n, s) \), depending only on \( n \) and \( s \).

In order to get our regularity result we first prove a maximum principle and then, using it, an interior and boundary regularity result for weak solutions of the problem.

As a consequence of our regularity result, along the paper we also deduce that the first eigenfunction of \( (-\Delta)^s \) is strictly positive in \( \Omega \).

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