

RESULTANT AND CONDUCTOR OF GEOMETRICALLY SEMI-STABLE SELF MAPS OF THE PROJECTIVE LINE OVER A NUMBER FIELD OR FUNCTION FIELD

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Abstract: We study the minimal resultant divisor of self-maps of the projective line over a number field or a function field and its relation to the conductor. The guiding focus is the exploration of a dynamical analog to Theorem 1.1, which bounds the degree of the minimal discriminant of an elliptic surface in terms of the conductor. The main theorems of this paper (5.5 and 5.6) establish that, for a degree 2 map, semi-stability in the Geometric Invariant Theory sense on the space of self maps, implies minimality of the resultant. We prove the singular reduction of a semi-stable presentation coincides with the simple bad reduction (Theorem 4.1). Given an elliptic curve over a function field with semi-stable bad reduction, we show the associated Lattès map has unstable bad reduction (Proposition 4.6). Degree 2 maps in normal form with semi-stable bad reduction are used to construct a counterexample (Example 3.1) to a simple dynamical analog to Theorem 1.1.

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Key words: Resultant, conductor, discriminant, bad reduction, critical bad reduction, Lattès map, minimality, self map of the projective line.