

**THIRD-POWER ASSOCIATIVE ABSOLUTE VALUED  
ALGEBRAS WITH A NONZERO IDEMPOTENT  
COMMUTING WITH ALL IDEMPOTENTS**

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**Abstract:** This paper deals with the determination of the absolute valued algebras with a nonzero idempotent commuting with the remaining idempotents and satisfying  $x^2x = xx^2$  for every  $x$ . We prove that, in addition to the absolute valued algebras  $\mathbb{R}$ ,  $\mathbb{C}$ ,  $\mathbb{H}$ , or  $\mathbb{O}$  of the reals, complexes, division real quaternions or division real octonions, one such absolute valued algebra  $A$  can also be isometrically isomorphic to some of the absolute valued algebras  $\mathbb{C}^*$ ,  $\mathbb{H}^*$ , or  $\mathbb{O}^*$ , obtained from  $\mathbb{C}$ ,  $\mathbb{H}$ , and  $\mathbb{O}$  by imposing a new product defined by multiplying the conjugates of the elements. In particular, every absolute valued algebra having the above properties is finite-dimensional. This generalizes some well known theorems of Albert, Urbanik and Wright, and El-Mallah.

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**Key words:** Absolute valued algebra, idempotent, division algebra, third-power associativity, pairwise commuting elements.