

## EMBEDDINGS OF LOCAL FIELDS IN SIMPLE ALGEBRAS AND SIMPLICIAL STRUCTURES

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**Abstract:** We give a geometric interpretation of Broussous–Grabitz embedding types. We fix a central division algebra  $D$  of finite index over a non-Archimedean local field  $F$  and a positive integer  $m$ . Further we fix a hereditary order  $\mathfrak{a}$  of  $M_m(D)$  and an unramified field extension  $E|F$  in  $M_m(D)$  which is embeddable in  $D$  and which normalizes  $\mathfrak{a}$ . Such a pair  $(E, \mathfrak{a})$  is called an embedding. The embedding types classify the  $\mathrm{GL}_m(D)$ -conjugation classes of these embeddings. Such a type is a class of matrices with non-negative integer entries. We give a formula which allows us to recover the embedding type of  $(E, \mathfrak{a})$  from the simplicial type of the image of the barycenter of  $\mathfrak{a}$  under the canonical isomorphism, from the set of  $E^\times$ -fixed points of the reduced building of  $\mathrm{GL}_m(D)$  to the reduced building of the centralizer of  $E^\times$  in  $\mathrm{GL}_m(D)$ . Conversely the formula allows to calculate the simplicial type up to cyclic permutation of the Coxeter diagram.

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**Key words:** Embeddings types, buildings, simple algebras, non-archimedean local fields.