THE RIESZ TRANSFORM, RECTIFIABILITY, AND REMOVABILITY FOR LIPSCHITZ HARMONIC FUNCTIONS

Fedor Nazarov, Xavier Tolsa, and Alexander Volberg

Abstract: We show that, given a set $E \subset \mathbb{R}^{n+1}$ with finite $n$-Hausdorff measure $\mathcal{H}^n$, if the $n$-dimensional Riesz transform

$$R_{\mathcal{H}^n}E f(x) = \int_E \frac{x - y}{|x - y|^{n+1}} f(y) \, \mathcal{H}^n(y)$$

is bounded in $L^2(\mathcal{H}^n | E)$, then $E$ is $n$-rectifiable. From this result we deduce that a compact set $E \subset \mathbb{R}^{n+1}$ with $\mathcal{H}^n(E) < \infty$ is removable for Lipschitz harmonic functions if and only if it is purely $n$-unrectifiable, thus proving the analog of Vitushkin’s conjecture in higher dimensions.

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