

THE RIESZ TRANSFORM, RECTIFIABILITY, AND REMOVABILITY FOR LIPSCHITZ HARMONIC FUNCTIONS

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Abstract: We show that, given a set $E \subset \mathbb{R}^{n+1}$ with finite n -Hausdorff measure \mathcal{H}^n , if the n -dimensional Riesz transform

$$R_{\mathcal{H}^n \llcorner E} f(x) = \int_E \frac{x - y}{|x - y|^{n+1}} f(y) \mathcal{H}^n(y)$$

is bounded in $L^2(\mathcal{H}^n \llcorner E)$, then E is n -rectifiable. From this result we deduce that a compact set $E \subset \mathbb{R}^{n+1}$ with $\mathcal{H}^n(E) < \infty$ is removable for Lipschitz harmonic functions if and only if it is purely n -unrectifiable, thus proving the analog of Vitushkin's conjecture in higher dimensions.

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