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MINIMAL FAITHFUL MODULES OVER ARTINIAN RINGS

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Abstract: Let R be a left Artinian ring, and M a faithful left R-module such that no proper submodule or homomorphic image of M is faithful.

If R is local, and socle(R) is central in R, we show that length$(M/J(R)M)$ + length(socle(M)) $\leq$ length(socle(R)) + 1.

If R is a finite-dimensional algebra over an algebraically closed field, but not necessarily local or having central socle, we get an inequality similar to the above, with the length of socle(R) interpreted as its length as a bimodule, and the summand $+1$ replaced by the Euler characteristic of a graph determined by the bimodule structure of socle(R). The statement proved is slightly more general than this summary; we examine the question of whether much stronger generalizations are possible.

If a faithful module M over an Artinian ring is only assumed to have one of the above minimality properties – no faithful proper submodules, *or* no faithful proper homomorphic images – we find that the length of $M/J(R)M$ in the former case, and of socle(M) in the latter, is $\leq$ length(socle(R)). The proofs involve general lemmas on decompositions of modules.

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Key words: Faithful modules over Artinian rings, length of a module or bimodule, socle of a ring or module.