A NONLOCAL 1-LAPLACIAN PROBLEM
AND MEDIAN VALUES

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Abstract: In this paper, we study solutions to a nonlocal 1-Laplacian equation given by

$$-\int_{\Omega_J} J(x-y)\frac{u_\psi(y)-u(x)}{|u_\psi(y)-u(x)|}\,dy = 0 \quad \text{for } x \in \Omega,$$

with $u(x) = \psi(x)$ for $x \in \Omega_J \setminus \overline{\Omega}$. We introduce two notions of solution and prove that the weaker of the two concepts is equivalent to a nonlocal median value property, where the median is determined by a measure related to J. We also show that solutions in the stronger sense are nonlocal analogues of local least gradient functions, in the sense that they minimize a nonlocal functional. In addition, we prove that solutions in the stronger sense converge to least gradient solutions when the kernel J is appropriately rescaled.

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