

NONLOCAL ELLIPTIC EQUATIONS IN BOUNDED DOMAINS: A SURVEY

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Abstract: In this paper we survey some results on the Dirichlet problem

$$\begin{cases} Lu = f & \text{in } \Omega \\ u = g & \text{in } \mathbb{R}^n \setminus \Omega \end{cases}$$

for nonlocal operators of the form

$$Lu(x) = \text{PV} \int_{\mathbb{R}^n} \{u(x) - u(x+y)\} K(y) dy.$$

We start from the very basics, proving existence of solutions, maximum principles, and constructing some useful barriers. Then, we focus on the regularity properties of solutions, both in the interior and on the boundary of the domain.

In order to include some natural operators L in the regularity theory, we do not assume any regularity on the kernels. This leads to some interesting features that are purely nonlocal, in the sense that they have no analogue for local equations.

We hope that this survey will be useful for both novel and more experienced researchers in the field.

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Key words: Integro-differential equations, bounded domains, regularity.