A MONOTONICITY FORMULA FOR MINIMAL SETS WITH A SLIDING BOUNDARY CONDITION

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Abstract: We prove a monotonicity formula for minimal or almost minimal sets for the Hausdorff measure $\mathcal{H}^d$, subject to a sliding boundary constraint where competitors for $E$ are obtained by deforming $E$ by a one-parameter family of functions $\varphi_t$ such that $\varphi_t(x) \in L$ when $x \in E$ lies on the boundary $L$. In the simple case when $L$ is an affine subspace of dimension $d-1$, the monotone or almost monotone functional is given by $F(r) = r^{-d}\mathcal{H}^d(E \cap B(x, r)) + r^{-d}\mathcal{H}^d(S \cap B(x, r))$, where $x$ is any point of $E$ (not necessarily on $L$) and $S$ is the shade of $L$ with a light at $x$. We then use this, the description of the case when $F$ is constant, and a limiting argument, to give a rough description of $E$ near $L$ in two simple cases.

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