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A MONOTONICITY FORMULA FOR MINIMAL SETS WITH A SLIDING BOUNDARY CONDITION

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Abstract: We prove a monotonicity formula for minimal or almost minimal sets for the Hausdorff measure $\mathcal{H}^d$, subject to a sliding boundary constraint where competitors for E are obtained by deforming E by a one-parameter family of functions φ_t such that $\varphi_t(x) \in L$ when $x \in E$ lies on the boundary L. In the simple case when L is an affine subspace of dimension $d-1$, the monotone or almost monotone functional is given by $F(r) = r^{-d}\mathcal{H}^d(E \cap B(x,r)) + r^{-d}\mathcal{H}^d(S \cap B(x,r))$, where x is any point of E (not necessarily on L) and S is the shade of L with a light at x. We then use this, the description of the case when F is constant, and a limiting argument, to give a rough description of E near L in two simple cases.

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Key words: Minimal sets, almost minimal sets, monotonicity formula, sliding boundary condition, Plateau problem.