

COMMON ZEROS PRESERVING MAPS ON VECTOR-VALUED FUNCTION SPACES AND BANACH MODULES

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Abstract: Let X, Y be Hausdorff topological spaces, and let E and F be Hausdorff topological vector spaces. For certain subspaces $A(X, E)$ and $A(Y, F)$ of $C(X, E)$ and $C(Y, F)$ respectively (including the spaces of Lipschitz functions), we characterize surjections $S, T: A(X, E) \rightarrow A(Y, F)$, not assumed to be linear, which jointly preserve common zeros in the sense that $Z(f - f') \cap Z(g - g') \neq \emptyset$ if and only if $Z(Sf - Sf') \cap Z(Tg - Tg') \neq \emptyset$ for all $f, f', g, g' \in A(X, E)$. Here $Z(\cdot)$ denotes the zero set of a function. Using the notion of point multipliers we extend the notion of zero set for the elements of a Banach module and give a representation for surjective linear maps which jointly preserve common zeros in module case.

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