

SOBOLEV REGULARITY OF THE BEURLING TRANSFORM ON PLANAR DOMAINS

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Abstract: Consider a Lipschitz domain Ω and the Beurling transform of its characteristic function $\mathcal{B}\chi_\Omega(z) = -\text{p.v.} \frac{1}{\pi z} * \chi_\Omega(z)$. It is shown that if the outward unit normal vector N of the boundary of the domain is in the trace space of $W^{n,p}(\Omega)$ (i.e., the Besov space $B_{p,p}^{n-1/p}(\partial\Omega)$) then $\mathcal{B}\chi_\Omega \in W^{n,p}(\Omega)$. Moreover, when $p > 2$ the boundedness of the Beurling transform on $W^{n,p}(\Omega)$ follows. This fact has far-reaching consequences in the study of the regularity of quasiconformal solutions of the Beltrami equation.

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