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ON VISCOSITY SOLUTIONS TO THE DIRICHLET PROBLEM FOR ELLIPTIC BRANCHES OF INHOMOGENEOUS FULLY NONLINEAR EQUATIONS

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Abstract: For scalar fully nonlinear partial differential equations $F(x, D^2u(x)) = 0$ with $x \in \Omega \Subset \mathbb{R}^N$, we present a general theory for obtaining comparison principles and well posedness for the associated Dirichlet problem, where $F(x, \cdot)$ need not be monotone on all of $\mathcal{S}(N)$, the space of symmetric $N \times N$ matrices. We treat *admissible viscosity solutions* u of *elliptic branches* of the equation in the sense of Krylov [**20**] and extend the program initiated by Harvey and Lawson [**11**] in the homogeneous case when F does not depend on x. In particular, for the set valued map Θ defining the elliptic branch by way of the differential inclusion $D^2u(x) \in \partial\Theta(x)$, we identify a uniform continuity property which ensures the validity of the comparison principle and the applicability of Perron's method for the differential inclusion on suitably convex domains, where the needed boundary convexity is characterized by Θ. Structural conditions on F are then derived which ensure the existence of an elliptic map Θ with the needed regularity. Concrete applications are given in which standard structural conditions on F may fail and without the request of convexity conditions in the equation. Examples include perturbed Monge–Ampère equations and equations prescribing eigenvalues of the Hessian.

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